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TITLE: AN ANALYSIS OF COMPACT HEAT EXCHANGER PERFORMANCE

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AN ANALYSIS OF COMPACT HEAT EXCHANGER PREFORMANCE

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ABSTRACT

Many cryogenic heat exchangers employ high-conductance metallic screens or perforated plates separated by insulating spacers normal to the fluid flow direction. The former insures a high rate of heat transfer between the fluid streams while the latter reduces longitudinal conduction. Packed-bed regenerators employing wire screens, perforated plates, or granular particles also have a similar structure.

In these exchangers, any individual plate or particle has very high thermal conductivity and is essentially at a single temperature. As a result, the temperature profile along the length consists of many steps, rather than a single continuous curve. Conventional analysis based on a continuous temperature profile tends to overestimate the efficiency of these exchangers.

Axial conduction down the bed, caused by finite contact or spacer resistance, further reduces the effectiveness. The ineffectiveness due to axial conduction adds to that due to finite number of plates and finite heat transfer coefficient.

A closed form expression is derived for the efficiency of a compact heat exchanger with given design N_{tu} where the exchanger consists of n layers of metallic screens, plates, or particle layers. It is observed that the effect of discrete temperature profile becomes significant when the per stage design N_{tu} exceeds about 0.5.

INTRODUCTION

Many counterflow heat exchanger concepts [1-4] utilize high conductance metallic screens or perforated plates separated by insulating spacers. The former insures a high rate of heat transfer between the fluid streams while the latter reduces longitudinal conduction. Packed bed regenerators made of metallic wire screens, perforated plates, or granular particles have a similar arrangement. The contact resistance between successive screens or particles serves to reduce axial conduction. The normal design method, which may be considered a sort of "continuum approach" assumes a uniform distribution of heat transfer area and a constant axial conduction parameter [5,6].

But, in practice, an individual plate or particle in these exchangers has very high internal conductivity and is essentially at a single temperature. For example, consider a heat exchanger running with gaseous helium and operating in the laminar-flow regime. Depending on the geometry, the Nusselt number is between 3 and 8. Using thermal conductivities of helium gas and stainless steel, we get a conservative estimate of the Biot number,

$$N_{Bi} = N_{Nu} \frac{\kappa_{He}}{\kappa_{SS}} < \frac{8 \times 0.15}{15} < 0.08 \tag{1}$$

At such low values of Biot number the plates may be considered to be isothermal. The temperature gradient is sustained by the insulating spacers or contact resistances. As a result, the temperature profile along the length of the exchanger bed consists of many steps, flat portions representing the metallic plates alternating with steep rises representing the insulating spacers or contact resistances. Approximating the stepped temperature profile with a continuous one tends to overestimate the efficiency of the exchanger.

The overall inefficiency of a heat exchanger can be related to 3 main sources:

I. finite heat transfer coefficient,

II. finite number of plates, screens, or particle layers, and

III. axial conduction caused by finite thermal resistance of insulating spacers or by finite contact resistance.

Other sources of inefficiency such as flow maldistribution or heat transfer from the surroundings are assumed to be small. It may be noted that even with infinite heat transfer coefficient and zero axial conduction, the effectiveness is less than unity solely due to the finite number of heat transfer members in the bed.

THEORY

Figure 1 gives a schematic and a typical temperature profile in a segmented counterflow heat exchanger made of a finite number of plates separated by insulating spacers. In this paper the word "plate" stands for a perforated plate, wire screen, particle layer, or any other metallic member that couples the two fluid streams and itself has a very high internal conductivity. Dimensionless temperatures have been illustrated in the insert in Figure 1.

Let T_k = temperature of the k^{th} plate, assumed uniform over the plate; $t_k^{(1)}$ = temperature of the "min" fluid when entering the k^{th} plate; and $t_k^{(2)}$ = temperature of the "max" fluid when leaving the k^{th} plate. The subscripts "min" and "max" and the superscripts (1) and (2) refer to the fluid streams with smaller and larger heat capacity flow rate ($\dot{m}c_p$) respectively. It is further assumed that there are two fictitious plates at temperatures:

$T_0 = t_1^{(1)}$ = inlet temperature of "min" fluid; and

$T_{n+1} = t_{n+1}^{(2)}$ = inlet temperature of "max" fluid, n being the total number of plates and $t_{n+1}^{(1)}$ and $t_1^{(2)}$ the respective exit temperatures.

Referring to Fig. 1, an energy balance over the control volume around the k^{th} plate gives:

$$C_{min}\left(t_k^{(1)} - t_{k+1}^{(1)}\right) + \left(\frac{\kappa A_c}{\ell}\right)_k \left(T_{k-1} - T_k\right) =$$

$$C_{max}\left(t_k^{(2)} - t_{k+1}^{(2)}\right) + \left(\frac{\kappa A_c}{\ell}\right)_{k+1} \left(T_k - T_{k+1}\right) \quad ; \qquad (1)$$

where κ = thermal conductivity of the insulating spacer,

A_c = cross sectional area of the spacer,

ℓ = length of the spacer between plates,

n = number of plates in the heat exchanger, and the subscript k refers to the spacer between the $(k-1)^{th}$ and k^{th} plates.

Equation (1) may be simplified by defining two dimensionless parameters,

$$\nu = \frac{C_{min}}{C_{max}} = \text{capacity rate ratio} \qquad (2a)$$

and

$$\lambda_k = \left(\frac{\kappa A_c}{\ell}\right)_k / nC_{min}$$

= axial conduction parameter for the spacer between $(k-1)^{th}$ and k^{th} plates. (2b)

It should be noted that

$$\lambda_1 = \lambda_{n+1} = 0 \qquad (2c)$$

because the two ends of the heat exchanger are adiabatic. Then

$$\nu\left(t_k^{(1)} - t_{k+1}^{(1)}\right) + n\,\nu\,\lambda_k\left(T_{k-1} - T_k\right) = t_k^{(2)} - t_{k+1}^{(2)} + n\,\nu\,\lambda_{k+1}\left(T_k - T_{k+1}\right) \qquad (3)$$

The inlet and exit fluid temperatures over a particular (k^{th}) plate, which is at a uniform temperature T_k, are related by the equations [7]:

$$t_{k+1}^{(1)} - T_k = \left(t_k^{(1)} - T_k\right) e^{-N_{tu}^{(1)}/n} \qquad (4a)$$

and

$$T_k - t_k^{(2)} = \left(T_k - t_{k+1}^{(2)}\right) e^{-N_{tu}^{(2)}/n} \qquad (4b)$$

$N_{tu}^{(1)}$ and $N_{tu}^{(2)}$ are the one-side N_{tu}'s of the respective sides and n is the total number of plates. The overall design N_{tu} is related to them as:

$$N_{tu}^{D} = \left(\frac{1}{N_{tu}^{(1)}} + \frac{\nu}{N_{tu}^{(2)}}\right)^{-1} \qquad (5)$$

The three sets of equations (3), (4a), and (4b) numbering 3n determine the 3n variables: T_k, $1 \le k \le n$; $t_k^{(1)}$, $2 \le k \le n+1$; and $t_k^{(2)}$, $1 \le k \le n$. The variables $t_1^{(1)}$ and $t_{n+1}^{(2)}$ are the known inlet temperatures of the two fluid streams.

The following dimensionless temperatures and temperature differences are defined to express the governing equations in dimensionless form.

$$\Theta_k = \frac{T_k - t_{n+1}^{(2)}}{t_1^{(1)} - t_{n+1}^{(2)}}; \quad \Delta_k = \Theta_{k-1} - \Theta_k = \frac{T_{k-1} - T_k}{t_1^{(1)} - t_{n+1}^{(2)}};$$

$$\tau_k^{(1)} = \frac{t_k^{(1)} - T_k}{t_1^{(1)} - t_{n+1}^{(2)}}; \quad \tau_k^{(2)} = \frac{T_k - t_{k+1}^{(2)}}{t_1^{(1)} - t_{n+1}^{(2)}} \qquad (6)$$

These dimensionless temperatures have been illustrated in the insert in Fig. 1. It may be observed that $\tau_k^{(1)}$ and $\tau_k^{(2)}$ are the temperature differences of the two fluid streams from that of the k^{th} plate at their respective inlets to the plate.

Thus $\tau_1^{(1)} = \Delta_1$ and $\tau_n^{(2)} = \Delta_{n+1}$.

Let
$$\beta_{1,2} = e^{-N_{tu}^{(1),(2)}/n} \tag{7}$$

Then Eqs. (3) and (4) reduce to:

$$\nu\left(\tau_k^{(1)} - \tau_{k+1}^{(1)} + \Delta_{k+1}\right) + n\,\nu\,\lambda_k \Delta_k = \left(\tau_k^{(2)} - \tau_{k-1}^{(2)} + \Delta_k\right) + n\,\nu\,\lambda_{k+1}\,\Delta_{k+1}\ ; \tag{8}$$

$$\tau_{k+1}^{(1)} = \Delta_{k+1} + \beta_1\,\tau_k^{(1)} \tag{9a}$$

and
$$\tau_{k-1}^{(2)} = \Delta_k + \beta_2\,\tau_k^{(2)} \tag{9b}$$

Eliminating $\tau_{k+1}^{(1)}$ and $\tau_{k-1}^{(2)}$ between Eqs. (8) and (9)

$$\left(1 - \beta_1\right)\nu\,\tau_k^{(1)} = \left(1 - \beta_2\right)\tau_k^{(2)} + n\,\nu\left(\lambda_{k+1}\,\Delta_{k+1} - \lambda_k \Delta_k\right)$$

or,
$$\tau_k^{(2)} = \mu\,\tau_k^{(1)} + \gamma_k\,\Delta_k - \gamma_{k+1}\,\Delta_{k+1} \tag{10}$$

where
$$\mu = \frac{1 - \beta_1}{1 - \beta_2}\nu \quad \text{and} \quad \gamma_k = \frac{n\,\nu\,\lambda_k}{1 - \beta_2} \tag{11}$$

The 3n linear algebraic equations (9a), (9b), and (10) completely determine the temperature field. In appendix A, the system is further reduced to a set of only n equations by eliminating the fluid temperatures $\tau_k^{(1),(2)}$. The coefficient matrix is in pentadiagonal form, which can be solved using standard algorithm [8]. Considering that n is a fairly large number, the computer solution of this set of equations is not trivial. Attempts to generate an analytical solution have been unsuccessful. The following approximate procedure, however, yields a convenient formula for the efficiency.

As discussed earlier, the efficiency of the heat exchanger can be ascribed to 3 sources: (I) finite heat transfer coefficient, (II) finite number of plates, and (III) axial conduction through the insulating spacers. An exact analytical model incorporating sources I and II yields a closed form solution for the efficiency ϵ_{I+II}. On eliminating the source I by setting the heat transfer coefficient $h \to \infty$, we obtain the efficiency ϵ_{II}. A second model including sources II and III yields the efficiency ϵ_{II+III}. For most cryogenic heat exchangers the inefficiencies are small and those from different sources can, in general, be added. Thus,

$$i_{I+II+III} = i_{I+II} + i_{II+III} - i_{II} \tag{12}$$

where i = inefficiency = 1 - ϵ. Equation (12) can also be written as:

$$\epsilon_{I+II+III} = \epsilon_{I+II} + \epsilon_{II+III} - \epsilon_{II} \tag{13}$$

HEAT EXCHANGER WITH FINITE NUMBER OF PLATES AND FINITE HEAT TRANSFER COEFFICIENT (I + II)

Because it is assumed that the spacers have infinite thermal resistance in the axial direction, the successive plates have no coupling between them. The overall exchanger may be considered to be made of n identical units operating in series, the design N_{tu} of each individual unit being N_{tu}^D/n. The efficiency of such a heat exchanger is given by Kays and London [7] as:

$$\varepsilon = \frac{1 - p^n}{1 - \nu\, p^n} \tag{14}$$

$$\text{with } p = \frac{1 - \varepsilon_k}{1 - \nu\, \varepsilon_k} \tag{15}$$

where ε_k is the efficiency of any individual exchanger composed of only one plate.

The governing equations for the k^{th} unit are Eqs. (9) and (10) with $\lambda = 0$.

Then $\tau_k^{(2)} = \mu\, \tau_k^{(1)}$ (16)

and

$$\varepsilon_k = \frac{t_k^{(1)} - t_{k+1}^{(1)}}{t_k^{(1)} - t_{k+1}^{(2)}} = \frac{(1 - \beta_1)\, \tau_k^{(1)}}{\tau_k^{(1)} + \tau_k^{(2)}} = \frac{(1 - \beta_1)\, \tau_k^{(1)}}{(1 + \mu)\, \tau_k^{(1)}} = \frac{1 - \beta_1}{1 + \mu} \tag{17}$$

Substituting this result in Eq. (15) and using Eq. (11)

$$p = \frac{\mu + \beta_1}{1 + \mu\, \beta_2} \tag{18}$$

Thus the efficiency of a segmented exchanger of n plates without axial conduction through the spacers is given as:

$$\varepsilon_{I+II} = \frac{1 - p^n}{1 - \nu\, p^n} \tag{14}$$

where $$p = \frac{\mu + \beta_1}{1 + \mu \beta_2} \text{ and } \mu = \frac{1 - \beta_1}{1 - \beta_2} \nu \quad .$$

Although the above derivation yields an expression for the efficiency, it does not provide the detailed temperature profile. Such an analysis is presented in Appendix B.

Special cases:

(A) Infinite heat transfer coefficient (II): $N_{tu}^{(1)} = N_{tu}^{(2)} \rightarrow \infty$.

In this limit $\beta_1 = \beta_2 = 0$, leading to $\mu = \nu$ and $p = \nu$.

Then

$$\varepsilon_{II} = \frac{1 - \nu^n}{1 - \nu^{n+1}} \tag{19}$$

(B) Balanced Flow: $\nu = 1$.

In the case of $\nu = 1$ we have $p = 1$, and Eqs. (14) and (19) are indeterminate. Using L'Hospital's rule Eq. (14) reduces to

$$\varepsilon_{I+II} = \frac{n\,(1-\beta_1)\,(1-\beta_2)}{n\,(1-\beta_1)\,(1-\beta_2) + (1-\beta_1\beta_2)} \tag{20}$$

and Eq. (19) reduces to

$$\varepsilon_{II} = \frac{n}{n+1} \tag{21}$$

(C) Balanced Design: $N_{tu}^{(1)} = N_{tu}^{(2)}$.

In this case, using Eq. (5)

$$N_{tu}^{(1)} = N_{tu}^{(2)} = (1 + \nu)\, N_{tu}^{D}$$

Let $\beta_o = e^{-(1+\nu)N_{tu}^D/n}$

Then Eqs. (14) and (18) reduce to

$$\varepsilon_{I+II} = \frac{1 - p^n}{1 - \nu p^n} \text{ with } p = \frac{\nu + \beta_o}{1 + \nu \beta_o} \tag{22}$$

In the event of balanced design and balanced flow:

$$\varepsilon_{I+II} = \frac{n(1-\beta_o)}{n(1-\beta_o) + (1+\beta_o)} \tag{23}$$

(D) Continuous heat exchanger: $n \to \infty$.

In the limit of large n,

$$\beta_1 = e^{-N_{tu}^{(1)}/n} \simeq 1 - \frac{N_{tu}^{(1)}}{n} \text{ and } \beta_2 = e^{-N_{tu}^{(2)}/n} \simeq 1 - \frac{N_{tu}^{(2)}}{n} \tag{24}$$

Substituting in Eqs. (11) and (18), and using Eq. (5), we get:

$$\mu = \frac{N_{tu}^{(1)}}{N_{tu}^{(2)}} \nu \quad \text{and} \quad p = 1 - \frac{1-\nu}{n} N_{tu}^D \simeq e^{-(1-\nu)N_{tu}^D/n} \tag{25}$$

Using Eq. (25), Eq. (14) gives:

$$\varepsilon_{I+II} = \frac{1 - e^{-(1-\nu)N_{tu}^D}}{1 - \nu e^{-(1-\nu)N_{tu}^D}} \tag{26}$$

Equation (26) is the familiar expression [7] for the efficiency of continuous counterflow heat exchanger.

HEAT EXCHANGER WITH FINITE NUMBER OF PLATES AND FINITE LONGITUDINAL CONDUCTION (II + III)

Since the N_{tu} of each plate is infinite on both sides, the fluid exit temperatures are equal to the corresponding plate temperatures.

$$\beta_1 = \beta_2 = 0; \mu = \nu \text{ and } \gamma_k = n \nu \lambda_k$$

From Eq. (9)

$$\tau_k^{(1)} = \Delta_k \text{ and } \tau_k^{(2)} = \Delta_{k+1} \tag{27}$$

Substituting in Eq. (10)

$$\Delta_{k+1} = \nu \Delta_k + n \nu \left(\lambda_k \Delta_k - \lambda_{k+1} \Delta_{k+1}\right) \tag{28}$$

Assuming that all the spacers are identical, i.e., $\lambda_k = \lambda$ for $2 \leq k \leq n$ and $\lambda_1 = \lambda_{n+1} = 0$,

Eq. (28) can be rewritten as:

$$\Delta_{k+1} = q \Delta_k; \; 2 \leq k \leq n - 1$$

with

$$\Delta_2 = q \Delta_1/(1 + n \lambda) \text{ and } \Delta_{n+1} = \nu \; (1+n \lambda) \Delta_n \tag{29}$$

where

$$q = \nu(1 + n \lambda)/(1 + \nu \; n \lambda); \tag{30}$$

Successive application of Eq. (29) gives

$$\Delta_k = q^{k-1} \Delta_1/(1+n \lambda); \; 2 \leq k \leq n \text{ with } \Delta_{n+1} = \nu \; q^{n-1} \Delta_1 \tag{31}$$

Substituting Eq. (31) into the relation $\sum_{k=1}^{n+1} \Delta_k = 1$ and solving,

$$\Delta_1 = \frac{1 - \nu}{1 - \nu^2 q^{n-1}} \quad \text{and} \quad \Delta_{n+1} = \nu \; q^{k-1} \Delta_1 = \frac{\nu q^{n-1} - \nu^2 q^{n-1}}{1 - \nu^2 q^{n-1}} \tag{32}$$

Then the efficiency $\varepsilon_{II+III} = 1 - \Delta_{n+1} =$

$$\frac{1 - \nu\, q^{n-1}}{1 - \nu^2 q^{n-1}} \quad \text{with} \quad q = \frac{\nu\,(1+n\lambda)}{1 + \nu\, n\lambda} \tag{33}$$

<u>Special cases</u>

(A) Zero axial conduction: $\lambda = 0$

In this limit $q = \nu$ and $\varepsilon_{II} = (1 - \nu^n)/(1 - \nu^{n+1})$.

This is the same result as Eq. (19).

(B) Balanced flow: $\nu = 1$

In the case of $\nu = 1$, $q = 1$ and Eqs. (33) and (19) are indeterminate. Using L'Hospital's rule, Eq. (33) reduces to:

$$\varepsilon_{II+III} = \frac{n\,(1+\lambda)}{1 + n\,(1+2\lambda)} \tag{34}$$

and Eq. (19) reduces to:

$$\varepsilon_{II} = \frac{n}{n+1}$$

(C) Continuous heat exchanger: $n \to \infty$.

In the limit $n \to \infty$, the spacer length ℓ approaches 0; but λ is still finite and may be defined as $\lambda = \kappa A_c/LC_{min}$, L being the overall length of the heat exchanger. Then $q \simeq 1 - (1-\nu)/n\lambda\nu$. Using the relation $\mathrm{Lim}\,(1+x)^{1/x} = e$ we get $q^n \simeq e^{-(1-\nu)/\lambda\nu} \simeq q^{n-1}$. Substituting this relation in Eq. (33) we get

$$\varepsilon_{II+III} = \frac{1 - \nu\, e^{-\frac{1-\nu}{\lambda\nu}}}{1 - \nu^2\, e^{-\frac{1-\nu}{\lambda\nu}}} \tag{35}$$

In the limit of $n \to \infty$ and $\nu = 1$, using L'Hospital's rule, Eq. (35) gives:

$$\varepsilon_{II+III} = \frac{1 + \lambda}{1 + 2\lambda} \simeq 1 - \lambda \qquad (36)$$

EFFICIENCY OF THE OVERALL EXCHANGER

Combining Eqs. (14), (19), and (33) by the relation (13), we get

$$\varepsilon_{I+II+III} = \frac{1 - p^n}{1 - \nu p^n} + \frac{1 - \nu q^{n-1}}{1 - \nu^2 q^{n-1}} - \frac{1 - \nu^n}{1 - \nu^{n+1}} \qquad (37)$$

with $p = \dfrac{\mu + \beta_1}{1 + \mu \beta_2}$; $q = \dfrac{\nu(1 + n\lambda)}{1 + \nu n \lambda}$ and $\mu = \dfrac{1 - \beta_1}{1 - \beta_2} \nu$

Similarly for balanced flow ($\nu = 1$) operation Eqs. (20), (21), and (34) give:

$$\varepsilon_{I+II+III} = \frac{n(1-\beta_1)(1-\beta_2)}{n(1-\beta_1)(1-\beta_2) + (1-\beta_1\beta_2)} + \frac{n(1+\lambda)}{1 + n(1 + 2\lambda)} - \frac{n}{n+1} \qquad (38)$$

In the case of balanced design $\beta_1 = \beta_2$, the efficiency is still given by Eqs. (37) or (38) except that

$$p = \frac{\nu + \beta_o}{1 + \nu \beta_o} \quad \text{with} \quad \beta_o = e^{-(1+\nu) N_{tu}^D / n}$$

The overall effectiveness can be reduced to an effective N_{tu} by the relation [7]

$$N_{tu}^{eff} = \frac{1}{1 - \nu} \ln \frac{1 - \nu \varepsilon}{1 - \varepsilon} \qquad (39)$$

Like ε, N_{tu}^{eff} is a function of N_{tu}^{D}, ν, λ, and n. In Fig. 2, N_{tu}^{eff} is plotted against N_{tu}^{D} for selected values of λ and n with $\nu = 1$. The ideal Kays and London formula [7] for $n \to \infty$ and $\lambda = 0$ is represented by the straight line with slope = 1. It may be observed from the curves with $\lambda = 0$ that the effect of finite number of plates becomes significant when the per stage N_{tu}^{D} ($= N_{tu}^{D}/n$) exceeds 0.5. If it is higher than about 2, the effective N_{tu} approaches the number of plates n, confirming that it is not possible to exceed an effective N_{tu} of 1 per each plate.

In the presence of axial conduction, the efficiency is further reduced as shown in Fig. 2. The result of Kroeger [5] for a continuous exchanger with λ = 0.01 is indistinguishable from our calculation with n = 1000 over the range of N_{tu}^{D} shown in the figure. It is seen from Fig. 2 that at high values of N_{tu}^{D} the effective N_{tu} is determined by n and λ.

In the limit of continuous exchanger ($n \to \infty$), $\varepsilon_{II} = 1$ and Eqs. (13), (26), and (35) give

$$\varepsilon_{I+III} = \frac{1 - e^{-(1-\nu)\, N_{tu}^{D}}}{1 - \nu\, e^{-(1-\nu)\, N_{tu}^{D}}} + \frac{1 - \nu\, e^{-\frac{(1-\nu)}{\lambda\nu}}}{1 - \nu^{2}\, e^{-\frac{(1-\nu)}{\lambda\nu}}} - 1 \tag{40}$$

which reduces to

$$\varepsilon_{I+III} = \frac{N_{tu}^{D}}{N_{tu}^{D} + 1} + \frac{1 + \lambda}{1 + 2\lambda} - 1 = \frac{N_{tu}^{D}}{N_{tu}^{D} + 1} - \frac{\lambda}{1 + 2\lambda} \tag{41}$$

for balanced flow ($\nu = 1$) operation. If the axial conduction parameter λ is sufficiently small, Eq. (41) and Eq. (20) of Ref. 5 yield the same values for the effectiveness.

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APPENDIX A

The governing equations may be summarized as:

$$\tau_{k+1}^{(1)} = \Delta_{k+1} + \beta_1 \tau_k^{(1)} \text{ with } \tau_1^{(1)} = \Delta_1 \tag{9a}$$

$$\tau_{k-1}^{(2)} = \Delta_k + \beta_2 \tau_k^{(2)} \text{ with } \tau_n^{(2)} = \Delta_{n+1} \tag{9b}$$

$$\text{and } \tau_k^{(2)} = \mu \, \tau_k^{(1)} + \gamma_k \Delta_k - \gamma_{k+1} \Delta_{k+1} \tag{10}$$

By successive application of Eqs. (9a) and (9b), respectively, we get

$$\tau_k^{(1)} = \sum_{i=1}^{k} \beta_1^{(k-i)} \Delta_i \tag{A1}$$

$$\text{and } \tau_k^{(2)} = \sum_{j=k+1}^{n+1} \beta_2^{(j-k-1)} \Delta_j \tag{A2}$$

Substituting these expressions in Eq. (10)

$$\sum_{j=k+1}^{n+1} \beta_2^{j-k-1} \Delta_j = \mu \sum_{i=1}^{k} \beta_1^{k-i} \Delta_i + \gamma_k \Delta_k - \gamma_{k+1} \Delta_{k+1} \; ; \; 1 \le k \le n \tag{A3}$$

Substituting k+1 for k in Eq. (A3),

$$\sum_{j=k+2}^{n+1} \beta_2^{j-k-2} \Delta_j = \mu \sum_{i=1}^{k+1} \beta_1^{k-i+1} \Delta_i + \gamma_{k+1} \Delta_{k+1} - \gamma_{k+2} \Delta_{k+2} \; ; \; 1 \le k \le n-1 \tag{A4}$$

And substituting k-1 for k in Eq. (A3),

$$\sum_{j=k}^{n+1} \beta_2^{j-k} \Delta_j = \mu \sum_{i=1}^{k-1} \beta_1^{k-i-1} \Delta_i + \gamma_{k-1} \Delta_{k-1} - \gamma_k \Delta_k \; ; \; 2 \le k \le n \tag{A5}$$

Multiplying Eq. (A4) by β_2, subtracting from Eq. (A3) and rearranging,

$$(1+\mu\beta_2)\Delta_{k+1} = \mu(1-\beta_1\beta_2)\sum_{i=1}^{k}\beta_1^{k-i}\Delta_i + \gamma_k\Delta_k - (1+\beta_2)\gamma_{k+1}\Delta_{k+1}$$

$$+\beta_2\gamma_{k+2}\Delta_{k+2} \; ; \; 1 \leq k \leq n-1 \quad \text{(A6)}$$

Substituting k for k+1 in Eq. (A6),

$$(1+\mu\beta_2)\Delta_k = \mu(1-\beta_1\beta_2)\sum_{i=1}^{k-1}\beta_1^{k-i-1}\Delta_i + \gamma_{k-1}\Delta_{k-1} - \gamma_k\Delta_k(1+\beta_2)$$

$$+\beta_2\gamma_{k+1}\Delta_{k+1} \; ; \; 2 \leq k \leq n \quad \text{(A7)}$$

On eliminating $\sum_{i=1}^{k-1}\beta_1^{k-i-1}\Delta_i$ between Eqs. (A6) and (A7), and rearranging,

$$b_k\Delta_{k-1} + c_k\Delta_k + d_k\Delta_{k+1} + e_k\Delta_{k+2} = 0; \; 2 \leq k \leq n-1 \quad \text{(A8)}$$

where

$$\begin{aligned} b_k &= \beta_1\gamma_{k-1} \\ c_k &= -[\mu + \beta_1 + (1+\beta_1+\beta_1\beta_2)\gamma_k] \\ d_k &= 1 + \mu\beta_2 + \gamma_{k+1}(1+\beta_2+\beta_1\beta_2) \\ e_k &= -\beta_2\gamma_{k+2} \end{aligned} \quad \text{(A9)}$$

On letting k = 1 in Eq. (A6) and rearranging,

$$c_1\Delta_1 + d_1\Delta_2 + e_1\Delta_3 = 0 \quad \text{(A10)}$$

where

$$\begin{aligned} c_1 &= -\mu(1-\beta_1\beta_2) \\ d_1 &= 1 + \mu\beta_2 + \gamma_2(1+\beta_2) \\ e_1 &= -\beta_2\gamma_3 \end{aligned} \quad \text{(A11)}$$

On multiplying Eq. (A5) with β_1, subtracting from Eq. (A3) and rearranging,

$$(1-\beta_1\beta_2)\sum_{j=k+1}^{n+1}\beta_2^{j-k-1}\Delta_j = (\mu+\beta_1)\Delta_k - \gamma_{k+1}\Delta_{k+1}$$

$$+ (1+\beta_1)\gamma_k\Delta_k - \beta_1\gamma_{k-1}\Delta_{k-1}; \; 2 \leq k \leq n \quad \text{(A12)}$$

which, for k = n, reduces to:

$$b_n\Delta_{n-1} + c_n\Delta_n = \Delta_{n+1} \quad \text{(A13)}$$

where, $b_n = -\dfrac{\beta_1\gamma_{n-1}}{(1-\beta_1\beta_2)}$ and $c_n = \dfrac{(\mu+\beta_1+\gamma_n+\beta_1\gamma_n)}{1-\beta_1\beta_2}$ (A14)

Defining $\Delta_k' = \Delta_k/\Delta_{n+1}$, Eqs. (A8), (A10), and (A13) can be rewritten as:

$$b_k\Delta_{k-1}' + c_k\Delta_k' + d_k\Delta_{k+1}' + e_k\Delta_{k+2}' = f_k \quad \text{(A15)}$$

Eqs. (A9), (A11), and (A14) define the constants b_k, c_k, d_k, and e_k.
Besides, $b_1 = d_n = e_n = 0$, $f_n = 1$, and $f_k = 0$ for $1 \leq k \leq n-1$ (A16)

From Eq. (A9) it should also be noted that $b_2 = e_{n-1} = 0$.
This set of equations may be solved for Δ_k' using the algorithm of Ref. 8. The complete temperature picture may be obtained by using the relations

$$\Delta_{n+1} = \left(1 + \sum_{k=1}^{n}\Delta_k'\right)^{-1} \text{ and } \Delta_k = \Delta_{n+1}\Delta_k'$$

APPENDIX B

HEAT EXCHANGER WITH FINITE NUMBER OF PLATES AND FINITE HEAT TRANSFER COEFFICIENT: TEMPERATURE PROFILE

Since axial conduction is absent

$$\gamma_k = 0; \; 1 \le k \le n+1 \tag{B1}$$

Substituting this condition in Eqs. (A8) and (A9)

$$\Delta_{k+1} = \frac{\mu + \beta_1}{1 + \mu \beta_2} \Delta_k = p \Delta_k \; ; \; 2 \le k \le n-1 \tag{B2}$$

Similarly from Eqs. (A10) and (A11)

$$\Delta_2 = \frac{\mu (1 - \beta_1 \beta_2)}{1 + \mu \beta_2} \Delta_1 = (p - \beta_1) \Delta_1 \tag{B3}$$

and from Eqs. (A13) and (A14)

$$\Delta_{n+1} = \frac{\mu + \beta_1}{1 - \beta_1 \beta_2} \Delta_n = \frac{\mu p}{p - \beta_1} \Delta_n \tag{B4}$$

Using the relation $\sum_{k=1}^{n+1} \Delta_k = 1$, Eqs. (B2-B4) give

$$\Delta_1 = \frac{p (1 - \nu)}{(\mu + \beta_1) (1 - \nu p^n)} \; ;$$

$$\Delta_k = p^{k-2} (p - \beta_1) \Delta_1 \quad ; \quad 2 \le k \le n$$

and
$$\Delta_{n+1} = p^{n-1} \mu \Delta_1 \tag{B5}$$

Eq. (B5) completely describes the temperature field.

The efficiency of the heat exchanger is now given as:

$$\epsilon_{I+II} = 1 - \tau_{n+1}^{(1)}$$

$$= 1 - \sum_{i=1}^{n+1} \beta_1^{n+1-i} \Delta_i \quad \text{(using Eq. (A1))}$$

$$= \left(1 - p^n\right) / \left(1 - \nu p^n\right). \tag{B6}$$

Eq. (B6) is the same as Eq. (14).

NOMENCLATURE

A_c = cross sectional area (m^2)

b_k, c_k, d_k, e_k, f_k = coefficients in governing equations

C = fluid heat capacity flow rate

= $\dot{m}c_p$ (W/K)

i = inefficiency

= $1 - \varepsilon$

L = overall length of heat exchanger (m)

ℓ = length of insulating spacer between two ... ssive plates (m)

n = number of plates in heat exchanger

N_{Bi} = Biot number

N_{Nu} = Nusselt number

N_{tu} = Number of heat transfer units

$p = (\mu + \beta_1)/(1 + \mu\beta_2)$

$q = \nu(1 + n\lambda)/(1 + \nu n\lambda)$

$t_k^{(1)}$ = temperature of the fluid with smaller heat capacity flow rate while entering the k^{th} plate (K)

$t_k^{(2)}$ = temperature of the fluid with larger heat capacity flow rate while leaving the k^{th} plate (K)

T_k = temperature of k^{th} plate (K) with $T_o = t_1^{(1)}$ and $T_{n+1} = t_{n+1}^{(2)}$

$\beta_o = e^{-(1+\nu)N_{tu}^{D}/n}$

$\beta_{1,2} = e^{-N_{tu}^{(1)(2)}/n}$

$\gamma_k = \nu n\lambda_k/(1 - \beta_2)$

$\Delta_k = (T_{k-1} - T_k) / (t_1^1 - t_{n+1}^2)$

$\Delta'_k = \Delta_k/\Delta_{n+1}$

ε = overall efficiency of heat exchanger

ε_k = efficiency of k^{th} plate

κ = thermal conductivity (W/mK)

λ_k = axial conduction parameter for the spacer between $(k-1)^{th}$ and k^{th} plates

= $(\kappa A_c/\ell)_k/\dot{m}c_{min}$ with $\lambda_1 = \lambda_{n+1} = 0$

λ = axial conductivity parameter when all spacers are identical

$\mu = \nu\ (1-\beta_1)\ /\ (1-\beta_2)$

ν = capacity rate ratio

= C_{min}/C_{max}

$\tau_k^{(1),(2)} = \pm\left(t_k^{(1)(2)} - T_k\right)\Big/\left(t_1^{(1)} - t_{n+1}^{(2)}\right)$ = dimensionless fluid temperature

$\Theta_k = \left(T_k - t_{n+1}^{(2)}\right)\Big/\left(t_1^{(1)} - t_{n+1}^{(2)}\right)$ = dimensionless plate temperature

Subscripts:

k : k^{th} plate

min : fluid stream with smaller heat capacity flow rate

max : fluid stream with larger heat capacity flow rate

I : finite heat transfer coefficient

II : finite number of plates

III : axial conduction

Superscripts:

D : design

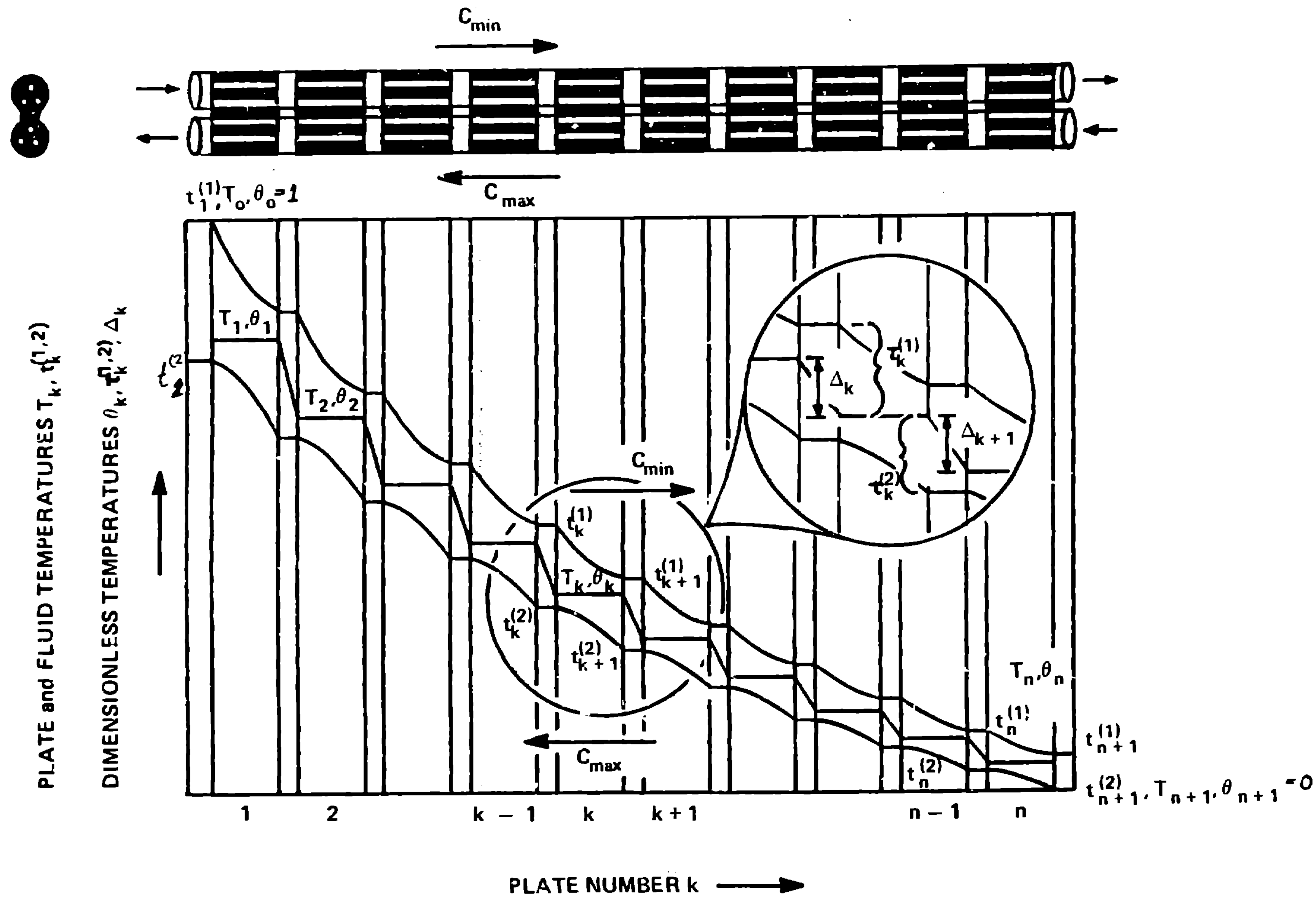
(1) : fluid stream with smaller heat capacity flow rate

(2) : fluid stream with larger heat capacity flow rate

FIGURE CAPTIONS

Fig. 1. Plate and fluid temperature profiles in a segmented heat exchanger. The insert illustrates dimensionless temperatures.

Fig. 2. The effective N_{tu} in a segmented heat exchanger shown against design N_{tu} for selected values of number of plates n, and axial conduction parameter λ. ——— $n \to \infty$, $\lambda = 0$; ----- $\lambda = 0$; -·-·-·- $\lambda = 0.01$. The curve for $n \to \infty$ and $\lambda = 0.01$ coincides with that for $n = 1000$, $\lambda = 0.01$ for this range of N_{tu}^D.



C min
C max
PLATE and FLUID TEMPERATURES T k , t k (1,2)
DIMENSIONLESS TEMPERATURES θ k , t k (1,2) , Δ k
T 1 , θ 1
T 2 , θ 2
T k , θ k
T n , θ n
Δ k
Δ k + 1
1
2
k − 1
k
k + 1
n − 1
n
PLATE NUMBER k

Fig. 1.

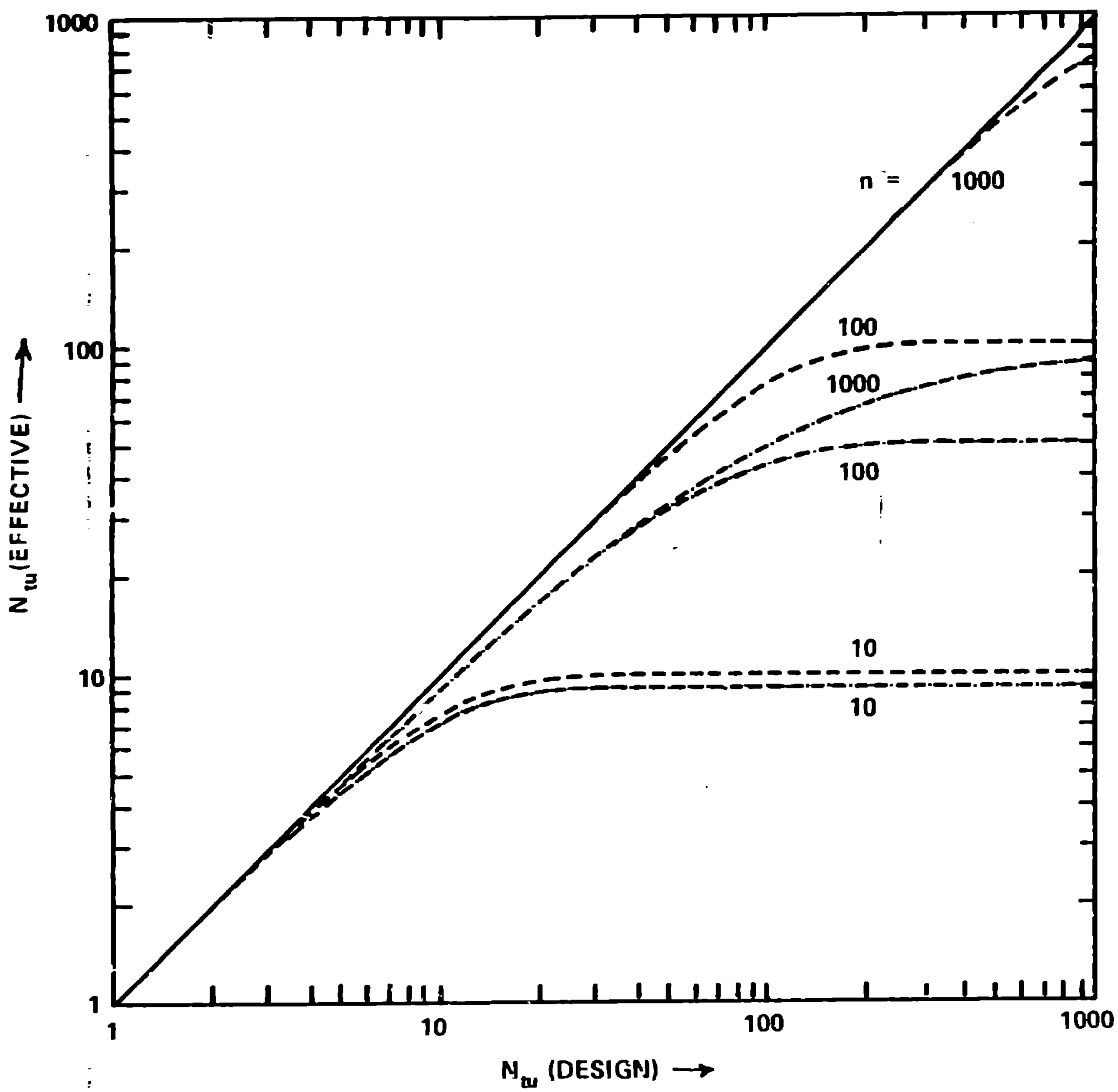

1000
100
10
1
N_{tu}(EFFECTIVE) →
1
10
100
1000
N_{tu} (DESIGN) →
n = 1000
100
1000
100
10
10

Fig. 2